

Use of Critical Speed Models from World Record Data to Estimate Limits of Human Ultra-Endurance Running Performance

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ABSTRACT

Ultra-endurance running (>42.2km) has increased in popularity in recent years and elite finish times continue to improve. Models of Critical Speed (CS) - the highest speed which can be maintained without fatigue - are routinely developed from short training distances to predict limits of performance in endurance events such as the marathon. **Purpose:** To model critical speed (CS) from current world record data to estimate limits of ultra-endurance running performance in males (M) and females (F). **Methods:** Current world record running performances from distances between 100m and 42.2km were used to develop two models of CS. A linear regression model used slope and y-intercept to estimate CS and finite anaerobic running distance (D'), respectively. Additionally, a curvilinear relationship between average speed and running distance was developed to estimate average speed at a given running distance. **Results:** For F, CS was determined to be 5.26 m·s⁻¹ and D' was 317.86 m. For M, CS was 5.82 m·s⁻¹ and D' was 345.48 m. This model predicted average speed from current world records could be maintained for 11.5% (M, 50km) to 68.3% (F, 24h run) greater distance. The curvilinear decay relationship between average speed and running distance underestimated average speed at 42.2km for both M and F, yet predicted 1.7% (M, 50km) to 28.2% (F, 24h run) greater average speed for ultra-endurance races. **Conclusions:** Current world records for ultra-endurance running are likely lesser than the physiological limits of humans and

the magnitude of this effect increases with greater race distance.

INTRODUCTION

Performance of elite endurance runners has improved substantially in recent years due to a complex interaction of financial incentives, high publicity, more efficient footwear, and better nutrition and training strategies. This is perhaps most evident at the marathon (42.2km) distance where highly publicized world record attempts and sub-2h attempts have been staged. Ultra-endurance races (running events > 42.2km) have gained popularity in recent years (9). Many ultra-endurance athletes compete in extreme environmental conditions, rugged terrain, and courses with challenging refueling logistics. Therefore, it is difficult to determine how current world records of ultra-endurance events compare to the limits of human performance. Despite the growth in popularity and consistent improvement in world record performances run in favorable conditions, the limits of human ultra-endurance running performance is unknown.

Previous researchers have used mathematical approaches to estimate limits of human performance during ultra-endurance running. Woodside predicted limits of human performance at distances up to 275km (the previous world record distance for 24h run)(13). However, the predicted performances from this model have been surpassed at both the

marathon and 161km (100 mile) distance. Using the improvement in world record running performances over the 20th century, Nevill and Whyte (7) predicted “peak” world record running speeds for males (M) and females (F) which have similarly been surpassed in recent years. More recently, Vandewalle developed critical speed (CS) models of elite runners from their personal best times at 1500 to 10000m to predict individual marathon running performance (12) and demonstrated that values of CS were similar when using different race distances to predict CS at 42.2km.

Critical speed (CS), the maximal running speed which can be sustained without fatigue, and the finite anaerobic running distance (D', that which can be run at greater than CS) are crucial determinants of performance in distance running (6). Models of CS are routinely developed for individual athletes from training data at various short training distances to predict performance in endurance events such as the marathon (10,12). A linear relationship exists between finish time (s) and distance (m) during maximal-effort running. The slope of this line represents CS and the y-intercept represents D'. Smyth et al. (10) recently demonstrated that models of CS using distances of 400-5000m had a very high $R^2 > 0.9999$ which also correlated well with marathon finish time. Furthermore, these authors demonstrated that elite runners were able to maintain speeds of approximately 93% of CS throughout the marathon distance. These data suggest that models of CS may be useful in estimating human performance during prolonged running.

Average running speed decays in a curvilinear manner with greater running distance (12). Thus, seen from a different lens, the asymptote of the distance-speed relationship may represent CS. CS models developed from linear regression or from a curvilinear decay model compare favorably with each other in predicting running performance in elite athletes (12). However, even in elite athletes, an asymptote is rarely demonstrated even with running distances up to 42.2km (12). Given the recent improvements in running performance at running distances ≥ 42.2 km which surpass previous models of the limits of human ultra-endurance running, this manuscript attempts to develop two models of CS from the fastest running performances in human history to predict limits of human ultra-endurance running.

METHODS

Data for world record finish times and distances for M and F runners were collected using publicly available sources on April 26, 2023. The university institutional review board approved this study as exempt from review due to the nature of the study (secondary analysis of publicly available data).

Due to a large contribution of acceleration to performance at very short distances, we excluded data from races < 100 m in distance. Specific race distances included: 100m, 200m, 400m, 800m, 1000m, 1500m, 1609m (1 mile), 2000m, 3000m, 5km (all surfaces), 10km (all surfaces), 1h run, 21.1km (half-marathon), 42.2km (marathon), 50km, 100km, 160.9km (100 mile), and 24h run. We noted a sharp drop in average pace between race distances of 42.2 and 50km in both M (-9.6%) and F (-11.7%). Furthermore, including distances > 42.2 km in linear regression analyses significantly weakened the R^2 value of both the linear CS model and curvilinear distance-speed relationship well beyond previous research values. Eliminating these distances resulted in R^2 values consistent with previous models of CS (10,12). Although races shorter than 1500m rely heavily on anaerobic energy production, eliminating these races from analysis had minimal ($< 0.04 \text{ m}\cdot\text{s}^{-1}$) effect on reducing estimates of CS, but considerably increased D' estimates ($\sim 50\%$) in both sexes. Therefore, we modeled CS for M and F runners using distances between 100m and 42.2km.

We developed two models (MS Excel) to predict conceptual limits of human performance in ultra-endurance running. In the first model, a linear regression line was fitted to data with the slope indicating CS and the y-intercept indicating D'. The regression model was used to model distance which could be covered if current world record finish times were run entirely at CS. In our second model, a curvilinear least squares model was fitted to data to determine average speed for each distance. Differences in average speed between sexes at each distance were calculated. Additionally, differences between actual and modeled average pace for ultra-endurance distances were calculated.

RESULTS

Figure 1 demonstrates the linear CS model for M and F runners. For F, the linear regression model demonstrated a CS of $5.26 \text{ m}\cdot\text{s}^{-1}$ and a D' of 317.9m ($R^2 = 0.9992$). For M runners, CS was determined to

be 5.82 m·s⁻¹ and D' of 345.5m (R² = 0.9994). Using this model, a finish time of 2h would allow M runners to achieve a distance of 42.26km. Furthermore, we calculated that finish times for 50km races should be able to be maintained for 55.78km (M) and 57.08km (F). For 24h running, this model predicted distance should be 57.5% (M) and 68.3% (F) greater than current world records.

Figure 2 demonstrates the curvilinear decay model of average pace at various running distances. Males displayed a greater average speed at all distances (mean: 11.6%, SD: 2.8%, range: 7.1-18.3%). For females, average speed between 100m and 42.2km decayed according to the function $y = 429.92 \times \text{distance (km)}^{-0.104}$ with R² = 0.9039. For males, average speed over this range decayed according to the function: $y = 479.73 \times \text{distance (km)}^{-0.103}$ with R² = 0.9249. At the 42.2km distance, this model average speed to be 7.4% (F) and 6.4% (M) slower than current world record running speed. However, at all ultra-endurance distances, our model predicted greater average speed than was observed in current world record performances. Moreover, for both M and F, the magnitude of difference between our model and the observed world record pace increased with increasing distance run.

DISCUSSION

Here, we estimate the limits of human performance in ultra-endurance running by adapting the concept of CS to current world record running performances. The major finding is that models of CS are substantially greater than average speed observed

in current world record performances for all ultra-endurance events in both M and F. Furthermore, the magnitude of this effect increases with increasing running distance. These data suggest that current world records in ultra-endurance events are well below the limits of human performance.

Relying on world record performances provides a unique opportunity to conceptualize the limits of ultra-endurance running performance. Although others have modeled limits of human performance in ultra-endurance running from elite athletes and historical records (7,12,13), many of the estimated limits of performance have been exceeded. Thus, our data provide an updated model of the limits of human ultra-endurance running performance. CS has previously been modeled from record performances of elite athletes on an individual level (8,12), but we believe this is the first attempt to model CS in a cross-sectional manner to estimate limits of human performance. The validity of this cross-sectional application of CS may be subject to scientific scrutiny. The cross-sectional approach used here relies on data from elite athletes with specialized disciplines. It is unlikely that elite endurance runners would be capable of achieving similar average speed of sprinters in short-duration events and vice versa. Notably, eliminating events of very short duration had minimal effect on measures of CS, but significantly increased estimates of D'. Therefore, we believe our approach reasonably approximates both CS and D'.

Our data do not include several middle- and long-distance records that remain to be ratified by governing bodies or were staged under non-

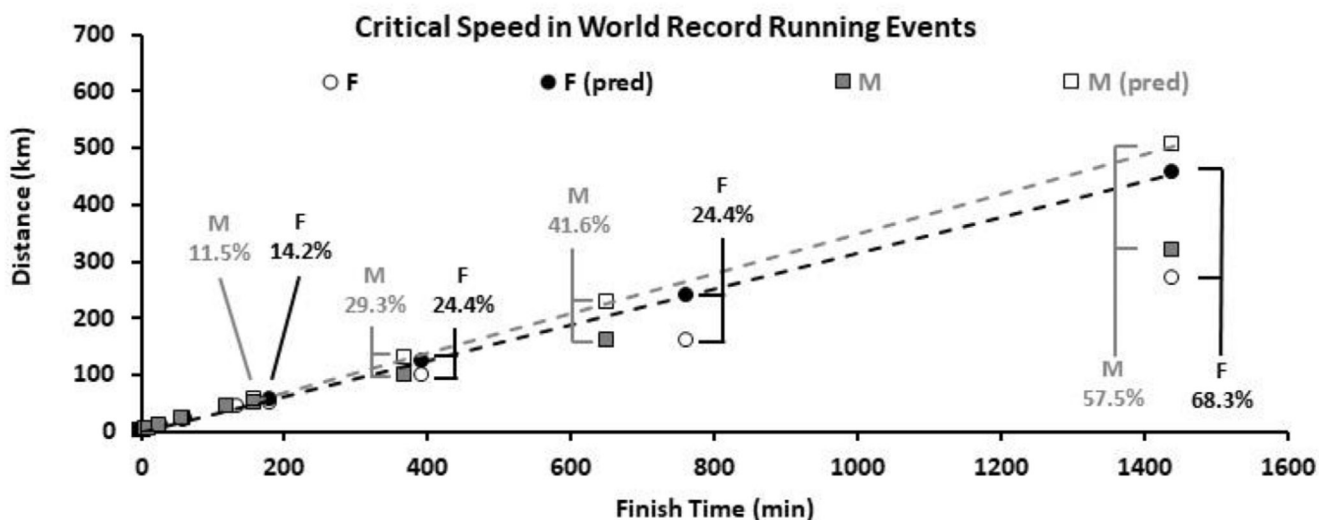


Figure 1. Linear Model of Human Critical Speed from World Record Running Performances. World record running performance data were collected from publicly available sources. Critical Speed (slope) and finite anaerobic running distance (D', y-intercept) were determined from linear regression using data from 100m-42.2km (dashed lines).

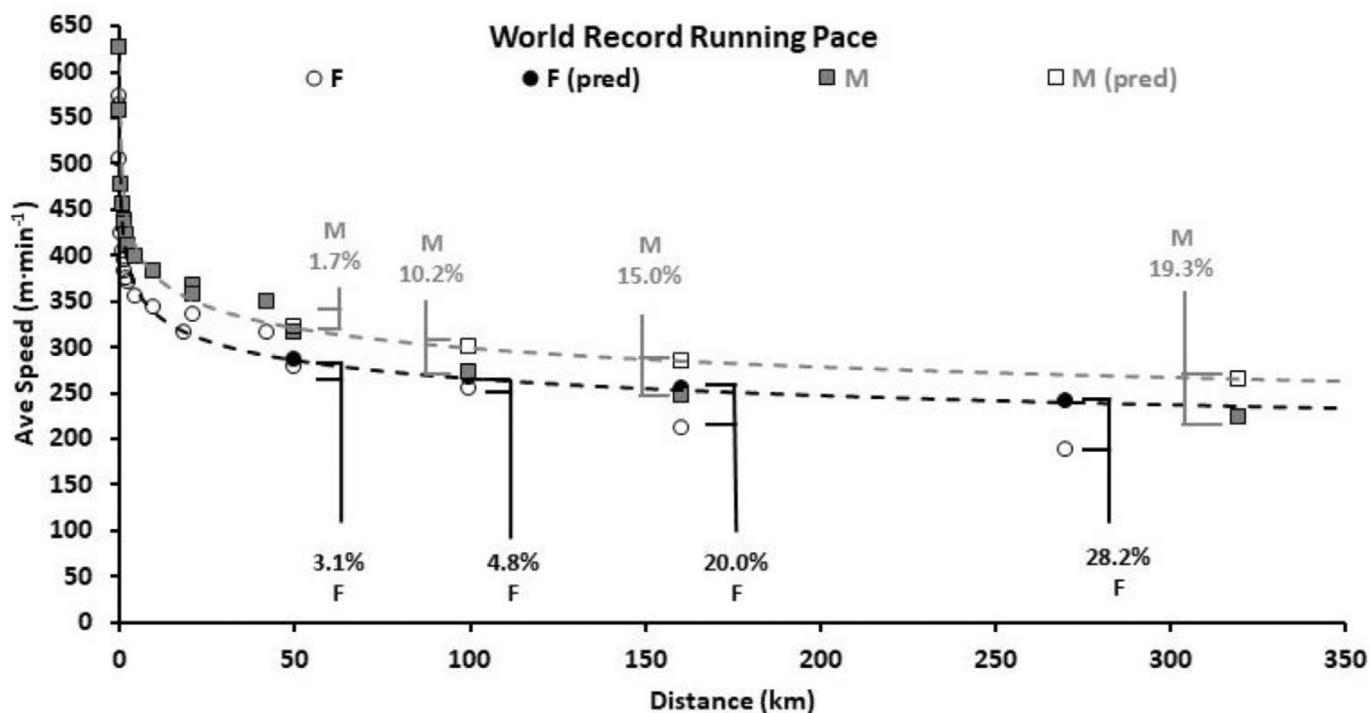


Figure 2. Model 2 of Critical Speed from World Record Running Performance. World record times from races 100m – 24h run were recorded. Average speed was calculated for each distance and a curvilinear model developed using data from 100m-42.2km (dashed lines). Percentage values display the difference between models and observed values.

world record eligible events. Given the large range of distances included in our model, it is unlikely that records set at middle-distance events will substantially alter our models of CS. However, records at longer distances such as 42.2km stand to have considerable impact on models of CS. Therefore, our model may underestimate absolute limits of performance, particularly at distances such as 50km.

Models of CS and D' will vary depending on which races are included. We limited our model to races between 100m and 42.2km. We chose this range to maintain the curvilinear distance-speed relationship and to retain linear regression $R^2 > 0.999$ which is consistent with previous work using CS for individual athletes. At very short running distances (i.e. 50m and 60m), average speed was lesser than those of races of 100m and 200m. We believe this indicates that acceleration is a stronger determinant of performance in these events than is average speed. Additionally, we noted that a large difference in average speed between 42.2 and 50km races in both M and F despite (ostensibly) similar metabolic and logistic factors between these race distances. Similarly, linear regression revealed a substantial decrease in R^2 when including longer races in our model of CS. Since ultra-endurance finish times are much greater than other race distances, including these races in a mathematical model may greatly reduce estimates of CS. Previous work has

demonstrated that CS calculated from 5km and 10km races corresponds well with CS during marathon running in elite athletes despite the large difference in running distance and finish time (10,12). Our models predict ultra-endurance running performance in events up to 24h in duration from data ranging up to 42.2km. The magnitude of difference between these distances is compares favorably to previous work that has estimated marathon running performance from 5km or 10km running.

Despite underestimating average pace at 42.2km by 6.4 (M) and 7.4% (F), our second model predicted average pace at 50km to be 1.7 (M) and 3.1% (F) greater than currently observed. Therefore, it is likely that substantial improvements could be made in improving running performance at this distance. This is consistent with previous models of ultra-endurance running performance (13). Notably, the magnitude of difference between our model and observed world record pace increased with greater distances. Although it is unclear whether CS can be maintained for the duration of ultra-endurance running events, elite runners appear to be able to maintain ~93% of CS during marathon running (10) and maximal aerobic speed occurs at approximately 86% of maximal oxygen uptake (VO_{2max}) (2). Therefore, we believe that elite runners may be able to maintain pace close to CS for the duration of at least some extreme ultra-endurance race distances.

There are many physiological and biomechanical differences that may predispose M or F to a competitive advantage in races at various distances (1). The current work provides a sex comparison for running performance at the elite level. The mean difference in average speed across all race distances was 11.5%. Interestingly lowest sex discrepancy was seen at the 100 mile race distance where average speed for M was 6.5% greater than F and the maximal sex difference was seen during the 24h race distance where average speed for M was 18.3% greater than F. Therefore, it appears that the effect of sex on average running speed is not influenced by running distance. Based on the current world record data, our CS models suggest M runners appear to have ~10% greater CS and D' than F.

The physiological mechanisms contributing to fatigue during ultra-endurance running is beyond the scope of the current study. However, average speed at longer running distances may be dictated by internal factors such as substrate utilization (5,11), hydration (4), muscle damage (3) as well as external factors such as competition logistics (refueling, running surface, terrain, etc). The relative contribution of these physiological determinants of ultra-endurance running is beyond the scope of the current manuscript but warrants further investigation.

PRACTICAL APPLICATION

Using current world record data to model CS, current ultra-endurance running performances appear to be slower than the limits of human running ability. M tend to display a 10% greater CS and D' than F. Pending world record performances suggest our models may underestimate the limits of human ultra-endurance performance for both sexes. It is likely that considerable improvements can be made in ultra-endurance running performance for both sexes.

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